CBSE Mathematics 2016 Solved paper for Class XII(10+2)

Section - A

Q. 1 For what value of k, the system of linear equations.

$$x + y + z = 2$$
$$2x + y - z = 3$$
$$3x + 2y + kz = 4$$

has a unique solution?

Ans. For any System of equations to have unique solution. |A|≠0

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} \neq 0$$

1 (k+2)-1(2k+3) + 1 (4-3)≠0

k+2-2k-3+1≠0

-k≠0

k≠0

Q. 2. If $\vec{a} = 4\vec{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\vec{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$

Ans. $\vec{a} + \vec{b} = 4\hat{i} - \hat{J} + \hat{k} + 2\hat{i} - 2\hat{j} + \hat{k}$

$\vec{a} + \vec{b}$ $= 6\hat{i} - 3\hat{j} + 2\hat{k}$

| $\vec{a} + \vec{b}$ | = $\sqrt{36 + 9 + 4} = \sqrt{49} = 7$

$Sol : \frac{1}{7}(6\hat{i} - 3\hat{j} + 2\hat{k})$

Q. 3 $Find\ \lambda\ and\ \mu\ if$

$\left(\hat{i}+3\hat{j}+9\hat{k}\right) X \left(3\hat{i}-\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}.$

Ans. $\hat{i}+3\hat{j}+9\hat{k}$ X $\left(3\hat{i}-\lambda\hat{j}+\mu\hat{k}\right)$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{bmatrix}=\vec{0}$$

$\hat{i}\left(3\mu+9\lambda\right)-\hat{j}\left(\mu-27\right)+\hat{k}\left(-\lambda-9\right)=0\hat{i}+0\hat{j}+0\hat{k}$

i.e. $3\mu+9\lambda=0$

$\mu-27=0 \Rightarrow [\mu=27]$

$-\lambda-9=0 \Rightarrow [\lambda=-9]$

Q. 4. Write the sum of intercepts cut off by the plane $\hat{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)-5=0$ on the three axes.

Ans. $2x+4-z=5$

$\frac{2}{5}x+\frac{4}{5}y-\frac{3}{5}z=1$

$\frac{\frac{x}{5}}{2}+\frac{4}{5}+\frac{3}{-5}=1$

Sum of intercepts cut offby plane on three axes $=\frac{5}{2}+5-5=\frac{5}{2}$

Q. 5 If. A= $\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find α satisfying o< α < $\frac{\pi}{2}$ when $A+A^T=\sqrt{2}I$ where A^T is transpose of A.

Ans. $A = \begin{bmatrix} \cos\alpha \sin\alpha \\ -\sin\alpha \cos\alpha \end{bmatrix}$

Condition given to us,

$$A + A^T = \sqrt{2}\, I_2$$

$$\begin{bmatrix} \cos a \sin a \\ -\sin a \cos a \end{bmatrix} + \begin{bmatrix} \cos d \sin a \\ \sin a \;\; \cos a \end{bmatrix} = \begin{bmatrix} \sqrt{2} \;\; 0 \\ 0\sqrt{2} \end{bmatrix}$$

$$2\cos\alpha = \sqrt{2}$$

$$\cos\alpha = \frac{1}{\sqrt{2}}$$

$$\left[\alpha = \frac{\Pi}{4}\right] 0 < \alpha < \frac{\Pi}{2}$$

Q. 6 if A is a 3X3 matrix and |3A| =k|A|, then write the value of K.

Ans. | 3 A |= k|A|

Using |KA|= $k^n \,|A|, Where\, is\, the\, order\, of\;\, square\, Matine\, \Delta$

$$|3A| = 3^3\,|A|$$
$$\Rightarrow (27\,|A| = k\,|A|)$$

k=27

SECTION B

Q. 7. Find $\int (x+3)\sqrt{3-4x-x^2dx.}$

Ans. $f\left(x+3\right)\sqrt{3-4x-x^2}$

$$x+3=\lambda\frac{d}{dx}\left(3-4x-x^2\right)+\mu$$

$$x+3=\lambda\left(-4-2x\right)+\mu$$

Comparing coefficients of x & constants

$$\left[\lambda=-\frac{1}{2}\right]$$

$$-4\lambda+\mu=3$$

$$2+\mu=3,\text{i.e.}\,\mu=1$$

$$\Rightarrow\int\left[-\frac{1}{2}(-4-2x)+1\right]\sqrt{3-4x-x^2}\;\;dx$$

$$-\frac{1}{2}\int\sqrt{t}\;dt\quad+\int\left(\sqrt{(\sqrt{7})^2}-(x+2)\right)^2dx,$$

$$Where\,t=-3-4x-x^2$$

$$\left(\Rightarrow-\frac{1}{2}\left(3-4x-x^2\right)^{3/2}+\frac{1}{2}\left(x+2\right)\sqrt{3-4x-x^2}\right)+\frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right)+c$$

Q. 8 Evaluate: $\int_{-2}^{2}\frac{x^2}{1+5^x}dx.$

$$Sol.\,f(x)=\int_{-2}^{2}\frac{x^2}{1+5^x}\;dx$$

$$f(-x) = \frac{x^2}{1+\frac{1}{5^x}}$$

$$= \frac{x^2 5^x}{1+5^x}$$

Using property

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a}\left[f(x)+f(-x)\right]dx$$

$$\int_{o}^{2}\left[\frac{x^2}{1+5^x}+\frac{x^2 5^x}{1+5^x}\right]dx \quad = \int_{0}^{2} x^2 dx = \frac{x^3}{3}\Big]_0^2$$

$= \frac{8}{3} Ans.$

Q. 9. Find the equation of tangents to the curve $y=x^3+2x-4$, which are perpendicular to line $x+14y+3=0$.

Ans.Let coordinates of point of contact be (x, y)as it lies on

$$y = x^3 + 2x - 4 \qquad -----A$$

$$\therefore y_1 = x_1^3 + 2x_1 - 4 \qquad --B$$

Differentiating A w.r.t. x

$$\frac{dy}{dx} = 3x^2 + 2$$

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 3x_1^2 + 2$$

Since the tangent at $(x_{1,} y_1)$is perpendicular to line $x+14y+3=0$

$\therefore$ slope of tangentat $(x_{1,} y_1)$ X Slope of line =-1

$$\left(3x_1^2+2\right)X\left(\frac{-1}{14}\right)=-1$$

$X_1 = \pm 2$

Now x_1=2 $\Rightarrow y_1 = 2^3 + 2x2 - 4 = 8$

$x_1 = -2 \Rightarrow y_1 = (-2)^3 + 2(-2) - 4$

$= -8 - 4 - 4$
$= -16$

Coordinates of point of contact are

$(2,8)$ & $(-2,-16)$

$$\left(\frac{dy}{dx}\right)_{(2,8)} = 3(2)^2 + 2 = 14$$

$$\left(\frac{dy}{dx}\right)_{(-2,-16)} = 3(-2)^2 + 2 = 14$$

Equationof tangent at (2,8)

y-8= 14 (x-2)

y =14x +36

Equiv of tangent at (-2,-16)

y+16 =14 (x+2)

y+16=14x +28

y =14x+12

Q. 10 If f(x)={ $\frac{\sin(a+1)x + 2\sin x}{x}$,x<0

2 ,x=0

$$\frac{\sqrt{1+bx}-1}{x} \quad ,x>0$$

is Continuous at x=0, then find the values of a and b

Ans. Since f (x) is continuous at x=0

$$\lim_{x\to 0-} f(x) = \lim_{x\to 0+} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0-} (a+1)\,\frac{\sin\,(a+1)x}{x(a+1)} + \lim_{x\to 0-}\ \frac{\sin\ x}{x} = 2$$

a+1+2=2

a=-1

$$\Rightarrow \lim_{x\to 0+} \frac{\sqrt{1+bx}-1}{x} \quad \frac{\sqrt{1+bx}+1}{\sqrt{1+bx}+1} = 2$$

$$\lim_{x\to 0+} \frac{1+bx-1}{x\left(\sqrt{1+bx}\ +1\right)} = 2$$

$$\frac{b}{2} = 2$$

b=4

Q. 11 .Solve for X: tan $^{-1}$ (x-1)+ tan $^{-1}$x+ tan $^{-1}$ (x+1) = tan $^{-1}$ 3x.

Ans. $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1) = \tan^{-1}3x$

$$\tan^{1}\frac{x-1+x+1}{1-x^2+1} + \tan^{-1}x = \tan^{-1}3x$$

$$\frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$$

$$\frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$$

$1+3x^2 = 2-x^2$

$4x^2 = 1$

$4x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

OR

Prove that $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-12x2}\right)=\tan^{-1}2x; |2x|<\frac{1}{\sqrt{3}}$.

$$\tan\text{–}^1\left(\frac{6x-8x^3}{1-12x^2}\right)\ \tan\text{–}^1\left(=\frac{4x}{1-4x^2}\right)=\tan^{-1}2x$$

$$\Rightarrow \tan^{-1}\frac{\frac{6x-8x^3}{1-12x^2}-\frac{4x}{1-4x^2}}{1+\left(\frac{6x-8x^3}{1-12x^2}\right)\left(\frac{4x}{1-4x^2}\right)}$$

$$\Rightarrow \tan^{-1}\frac{6x-8x^3-24x^3+32x^5-4x+48x^3}{1-4x^2-12x^2+48x^4+24x^2-32x^4}$$

$$\Rightarrow \tan^{-1}\frac{32x^5+16x^3+2x}{1+16x^4+8x^2}\Rightarrow \tan^{-1}2x$$

Q. 12 if x cos (a+y) = cosy then prove that .$\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$

Hence show that sina $\frac{d^2y}{dx^2}+\sin 2(a+y)\frac{dy}{dx}=0.$

Sol. $-x\sin(a+y)\frac{dy}{dx}+\cos(a+y)=-\sin y\frac{dy}{dx}$

$$\cos\,(a+y)=\frac{dy}{dx}\left[-\sin y+x\sin(a+y)\right]$$

$$\cos\,(a+y) = \frac{dy}{dx}\left[-\sin y + \frac{\cos y}{\cos(a+y)}\sin(a+y)\right]$$

$$\cos^2(a+y) = \frac{dy}{dx}\left[-\sin y\cos(a+y) + \cos y\sin(a+y)\right]$$

$$\frac{\cos^2(a+y)}{\sin(a+y-y)} = \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

Or

Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$

Sol. Put2x = $\sin\theta$

Sin$^{-1}\left(\frac{3\sin\theta - 4\cos\theta}{5}\right)$

$$\sin^{-1}\left(\frac{3}{5}\sin\theta - \frac{4}{5}\cos\theta\right)$$

$$\Rightarrow \sin^{-1}\left(\cos d\ \sin\theta - \sin d\cos\theta\right)\left[\begin{matrix}consider \\ \sin d = \frac{3}{5}\end{matrix}\right]$$

$$\Rightarrow \sin^{-1}\ \sin(\theta - d)$$

$$y = \theta - d$$

$$y = \sin^{-1}2x - \sin^{-1}\frac{1}{3}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

Q. 13 A bag X contains 4 white balls and 2 black balls, while another bag y contains 3 white balls and 3 black balls. Two balls are drawn (Without replacement) at random from of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag y.

Ans. E_1 = Select Bag X

E_2= Select Bag Y

A= Selecting 1 white & 1 black

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P(\frac{A}{E_1}) = \frac{{}^4c_1 X\ {}^2c_1}{{}^6c_2} = \frac{8}{15}$$

$$P(\frac{A}{E_2}) = \frac{{}^3c_1 X\ {}^3c_1}{{}^6c_2} = \frac{3}{5}$$

P(Balls drawn are from bag Y)= $P(\frac{E_2}{A}) = \dfrac{p(E_2)P(\frac{A}{E_2})}{p(E_1)P(\frac{A}{E_1}) + p(E_2)P(\frac{A}{E_2})}$

$$= \frac{1}{2} * \frac{3}{5} / \ (\frac{1}{2} * \frac{8}{15} + \frac{1}{2} * \frac{3}{5})$$

$$= \frac{\frac{3}{10}}{\frac{8}{30} + \frac{3}{10}} = \frac{\frac{3}{10}}{\frac{8+9}{30}} = = \frac{3x3}{17} = \frac{9}{17} ans.$$

Or

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning. If A starts first

Sol.

$E =$ *person A gets a* 10

$F =$ *person B gets a* 10

$$P(E) = \frac{3}{36} = \frac{1}{12}$$
$$p(\bar{E}) = \frac{11}{12}$$

$$P(F) = \frac{1}{12}$$
$$p(\bar{F}) = \frac{11}{12}$$

A wins if he throws10 in 1st, 3rd&5th throws............

a. $'10' \; in \; 1st \; throw = \frac{1}{12}$

b. '10' in 3rd throw = $P\left(\bar{E} \cap \bar{F} \cap E\right) = P(\bar{E})P(\bar{F})P(E)$

$$= \frac{11}{12} * \frac{11}{12} * \frac{1}{12} = (\frac{11}{12})^2 * \frac{1}{12}$$

c. '10' in 5th throw = $P\left(\bar{E} \cap \bar{F} \cap \bar{E} \cap \bar{F} \cap E\right)$

$$= \frac{11}{12} * \frac{11}{12} * \frac{11}{12} * \frac{11}{12} * \frac{1}{12}$$

$= = \left(\frac{11}{12}\right)^4 * \frac{1}{12}$ And so on............

(Probability of A wins)=

$$\Rightarrow P\left[E \cup \left(\bar{E} \cap \bar{F} \cap E\right) \cup (\bar{E} \cap \bar{F} \cap \bar{E} \cap \bar{F} \cap E) \cup ....\right]$$

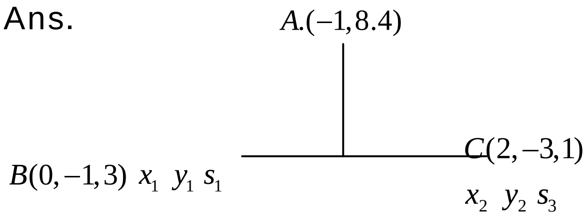
$$= \frac{1}{12} + \left(\frac{11}{12}\right)^2 * \frac{1}{12} + \left(\frac{11}{12}\right)^4 * \frac{1}{12} + ...$$

$$\Rightarrow \frac{\frac{1}{12}}{1 - \left(\frac{11}{12}\right)^2} = \frac{\frac{1}{12}}{1 - \frac{121}{144}} = \frac{1}{\not{12}} X \frac{\not{144}^{12}}{23} = \frac{12}{23}$$

P (B wins) $=1-\frac{12}{23}=\frac{11}{13}$

Q. 14 Find the coordinates of the foot of perpendicular drawn from the point A (-1, 8, 4) to the line joining the points B (0,-1, 3) and C (2, -3,-1). Hence find the image of A in line BC.

Ans.



Equation of BC $\frac{x-0}{2-0}=\frac{y+1}{-3+1}=\frac{z-3}{-1-3}$

$$\frac{x}{2}=\frac{y+1}{-2}=\frac{z-3}{-4}=\lambda$$

$$Coordinate\ of\ D\ (2\lambda,-2\lambda-1,-4\lambda+3)$$

$$Direction\ Ratios\ of\ \ AD\left(2\lambda+1,-2\lambda-9,-4\lambda-1\right)$$

$$S\ \text{in}\ ce\ \ AD\ is\ preperdicular\ toBC$$

$$2(2\lambda+1)-2(-2\lambda-9)-4(-4\lambda-1)=0$$

$$4\lambda+2+4\lambda+18+16\lambda+4=0$$

$$24\lambda+24=0$$

$$\lambda=-1$$

$$Coordinates\ of\ D(-2,1,7)$$

Image of A in line BC=

$$\frac{-1+x_1}{2}=-2\Rightarrow\ x_1=-3$$

$$\frac{8+y_1}{2}=1 \Rightarrow y_1=-6$$

$$\frac{4+z_1}{2}=7 \Rightarrow z_1=10$$

Image ofA in line BC is (-3,-6,10)

Q15.Show that the four points (0,-1,-1),(-4,4,4), (4,5,1) and (3, 9, 4)are coplanar. Find the equation of the plane containing them.

SOLUTION The equation of plane passing through (0,-1-1) is

$a(x-0)+b(y+1)+c(z+1)=0$ …………………i

If it passesthrough (-4, 4, 4) and (4, 5, 1) than

$a(-4)+b(5)+c(5)=0$ …………..ii

and, $a(4)+b(6)+c(2)=0$

or, $a(2)+b(3)+c(1)=0$

Solving ii and iii by cross multiplication, we obtain

$$\frac{a}{5-15}=\frac{b}{10+4}=\frac{c}{-12-10}$$

$$\Rightarrow \frac{a}{-5}=\frac{b}{7}=\frac{c}{-11}=\lambda(say)$$

$$\Rightarrow a=-5\lambda, b=7\lambda \; and \; c=-11\lambda$$

Substituting the values of a, b and c in i, we get

$$-5\lambda x+7\lambda(y+1)-11\lambda(z+1)=0$$

$$5x-7y+11z+4=0$$

Q. 16. A typist charges Rs 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are Rs. 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only Rs. 2 per from a poor student shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem?

Sol.Let the charge of typing one englishpage = Rs. x

the change of typing one hindipage =Rs. y

10x+3y=145

3x +10y=180

$\Lambda X = B$

Where $\begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 145 \\ 180 \end{bmatrix}$

$$|A| = 100 - 9 = 91 \neq 0$$

given system has unique solution

given by x= $A^{-1}B$

adj $A = \begin{bmatrix} 10-3 \\ -3\ 10 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} = adjA = \frac{1}{91} \begin{bmatrix} 10 & -3 \\ -3\ 10 & \end{bmatrix}$$

$$X = \frac{1}{91} \begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix} \begin{bmatrix} 145 \\ 180 \end{bmatrix}$$

$$= \frac{1}{91} \begin{bmatrix} 10*145 - 3*180 \\ -3*145 + 10*180 \end{bmatrix}$$

$$= \frac{1}{91}\begin{bmatrix} 910 \\ 1385 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

Charge for typing 1 English page = Rs.10

Charge for typing 1 Hindi page = Rs.15

Shyam was charged Rs.13 less than usual.

Q. 17. Find the particular solution of the differential equation.

$2y\, e^{x/y}\, dx+(y-2x\, e^{x/y})\, dy =0$

Given that x=0 when y=1.

Sol.We have,

$$2ye^{x/y}dx + (y - 2xe^{x/y})dy = o$$

$$\Rightarrow \frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}}$$

Clearly, the given differential equation is a homogeneous differential equation. As the right hand side of as a function of $\frac{x}{y}$ So, we put x=vyand $\frac{dx}{dy} = v + y\frac{dv}{dy}$ to get

$$v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$$

$$\Rightarrow y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v$$

$$\Rightarrow y\frac{dv}{dy} = -\frac{1}{2e^v}$$

$$\Rightarrow 2ye^v \; dv = -dy$$

$$2ye^v dv = -\frac{1}{y}dy$$

$$\Rightarrow 2\int e^{v} dv = -f\ \frac{y}{1} dy$$

$$\Rightarrow 2e^{v} = -\log|y| + \log C$$

$$\Rightarrow 2e^{v} = \log\left|\frac{c}{y}\right|$$

$$\Rightarrow 2e^{x/y} = \log\left|\frac{c}{y}\right|$$

$Hence, 2e^{x/y} = \log\left|\frac{c}{y}\right|$ given the general solution of the given different equation.

$$2e^{x/y} = \log\left|\frac{c}{y}\right|$$

x=0, y =1

2e ^{0} = log $\left(\frac{c}{1}\right)$

2=log C

$$2e^{x/y} = 2 - \log y$$
is the particular solution

Q. 18. Find the particular solution of differential equation $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$ given that y = 1 when x =0.

SOLUTION We have ,

$$\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x} \Rightarrow \frac{dy}{dx} + \frac{\cos x}{1 + \sin x} y = -\frac{x}{1 + \sin X}$$

This is a linear differential equation with P= $\frac{\cos x}{1 + \sin x}$ *and* $Q = \frac{-x}{1 + \sin X}$

I.F.=e $e\int \frac{\cos x}{1+\sin x}dx = e^{\log(1+\sin x)} = (1+\sin x)$

Multiplying both sides of i by I.F. = 1 + sin x, we get

(1+ sin x) $\frac{dy}{dx} + y\cos x = -x$

Integrating with respect to x, we get

$y(1+\sin x) = \int -x\,dx + C$

$\Rightarrow y(1+\sin x) = -\frac{x^2}{2} + C$

$\Rightarrow y = \frac{2C - x^2}{2(1+\sin X)}$, Which gives the generalsolution.

Put y=1 and x=0 , c=1

$\Rightarrow y = \frac{2 - x^2}{2(1+\sin X)}$

Q. 19 $Find: \int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

Ans.

$Put\; 2x = t, 2dx = dt$

$\frac{1}{2}\int \frac{(t-5)e^t}{(t-3)^3}dt$

$1/2\int \frac{[t-3]}{(t-3)^3} - \frac{2}{(t-3)^3}\; e^t dt$

$\frac{1}{2}\left[\int e^t \frac{1}{(t-3)^2}dt + \int \frac{-2e^t dt}{(t-3)^3}\right]$

$$\frac{1}{2}\left[\frac{e^t}{(t-3)^2}dt+\int\frac{2e^t dt}{(t-3)^3}-\int\frac{2et\ dt}{(t-3)^3}\right]$$

$$\frac{e^t}{2(t-3)^2}+c=\frac{e^{2x}}{2(2x-3)^2}+c$$

Or

$$Find:\int\frac{x^2+x+1}{\left(x^2+1\right)\left(x+2\right)}dx$$

Sol: let $\frac{x^2+x+1}{(x^2+1)(x+2)}=\frac{Ax+B}{x^2+1}+\frac{C}{x+2}$

$$x^2+x+1=Ax^2+2Ax+Bx+2B+Cx^2+C$$

Comparing cofficients of x^2, x & *cons tan ts*

$1=A+c$

$1 = 2A + B$

$1 = 2B + c$

$$A=\frac{2}{5}$$

$$\frac{3}{5}=C$$

$$B=\frac{1}{5}$$

$$\int\frac{\left[\frac{2}{5}x+\frac{1}{5}\right]}{x^2+1}dx+\int\frac{\frac{3}{5}\ dx}{x+2} = \frac{1}{5}\log(x^2+1)+\frac{1}{5}\tan^{-1}x+\frac{3}{5}\log(x+2)+c$$

SECTION-C

Q. 20 Prove that F $(\theta)=\frac{4\sin\theta}{2+\cos\theta}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$

Solution.we have,

$$f(\theta)=\frac{4\sin\theta}{2+\cos\theta}-\theta$$

$$\Rightarrow f'(\theta)=\frac{(2+\cos\theta)(4\cos\theta)+\sin^2\theta}{(2+\cos\theta)^2}-1$$

$$\Rightarrow f'(\theta)=\frac{8\cos\theta+4}{(2+\cos\theta)^2}-1$$

$$\Rightarrow f'(\theta)=\frac{4\cos\theta-\cos^2\theta}{(2+\cos\theta)^2}-1$$

$$\Rightarrow f'(\theta)=\frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}.0\, for\, all\, \theta\in\left(0,\frac{\pi}{2}\right).[\because \cos\theta>0, 4-\cos\theta>0 and 2+\cos\theta>0]$$

Hence, $\Rightarrow f'(\theta) is incear\sin g\ on\left(0,\frac{\pi}{2}\right).$

OR

show that the semi- vertical angle of a cone of maximum volume and given slant height is

$$\tan^{-1}\sqrt{2}\ \ or\cos^{-1}\frac{1}{\sqrt{3}}.$$

Solution let a be the semi show that the semi- vertical angle of a cone VAB of given slant height t. in AOV,

$\cos a = \frac{VO}{VA} and \sin a = \frac{OA}{VA}$

$\Rightarrow \cos a = \frac{VO}{l} and \sin a = \frac{OA}{l}$

$\Rightarrow VO = l\cos a, OA = l\sin a$

Let be the volume of the cone, then,

$\Rightarrow V = \frac{1}{3}\pi(OA)^2(VO)$

$\Rightarrow V = \frac{1}{3}\pi(l\sin a)^2\ l\cos a$

$\Rightarrow V = \frac{1}{3}l^3\left(-\sin^3 a + 2\sin a\cos^2 a\right)$

The critical points of are given by $\frac{dV}{da} = 0$

Check RD Sharma part 1 XII for further solution.

Q. 21 Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+y)^2 \end{vmatrix} = 2xyz\left(x+y+z\right)^3$$

Or $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

Solution

$$\Delta = 1/abc\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} Multiplying R_1, R_2 and R_3 by a, b, and c$$

Respectively, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & (c+a)^2 b & cb^2 \\ ac^2 & bc^2 & (a+b)^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc}abc\begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & (c+a)^2 b & cb^2 \\ ac^2 & bc^2 & (a+b)^2 \end{vmatrix}\left[Taking\ a,b\ and\ c\ common\ from\ c_{1,} c_2\ and c_3 respectively\right]$$

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} Appliying C_1 \to C_2 - C_3 and\ C_2 \to C_2 - C_3$$

we get $\Delta = \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (c+a)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$

$$\Rightarrow \Delta = (a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix} Taking\ (a+b+c) common\ From\ C_1\ \&\ C_2$$

$$\Rightarrow \Delta = (a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix} Appliying\ R_3 \to R_3 - (R_1 + R_2)$$

OR $\Rightarrow \Delta = 2abc(a+b+c)^3$

If A = $\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{vmatrix}$ $and\ A^3 - 6A^2 + 7A + KI_3 = 0 Find\ k.$

Sol.

$$A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \quad A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

K=2

Q. 22. Using the method of integration, find the area of the triangular region whose vertices are A(2, -2), B(4,3) and C(1,2)

Ans.Equation of AB

$$Y + 2 = \left(\frac{3+2}{4-2}\right)(x-2)$$

$$Y + 2 = \left(\frac{5}{2}\right)(x-2)$$

$$Y = \left(\frac{5}{2}\right)x - 7 \Rightarrow x = (y+7)\frac{2}{5}$$

eqn of BC

$$y - 3 = \left(\frac{2-3}{1-4}\right)(x-4)$$

$$y - 3 = \frac{\cancel{-}1}{\cancel{-}3}(x-4)$$

$$y=\frac{1}{3}x+\frac{5}{3}\Rightarrow\left(y-\frac{5}{3}\right)3=x$$

equl of AC

$$Y+2\left(\frac{2+2}{1-2}\right)(x-2)$$

$$Y+2=\frac{4}{-1}(x-2)$$

$$y=-4x+6\Rightarrow\frac{y-6}{-4}=x$$

$$\int_1^2(BC-AC)dx+\int_2^4(BC-AB)dx$$

$$\frac{1}{3}\int_1^2(x+5+12x-18)dx+\int_2^4\left[\frac{x+5}{3}-\frac{5}{2}x+5\right]dx$$

$$\frac{1}{3}\int_1^2(13x-13)dx\quad+\frac{1}{6}\int_2^4 2x+10-15x+42dx$$

$$\frac{1}{3}\left[13\frac{x^2-13x}{2}\right]_1^2+\frac{1}{6}\left[-13\frac{x^2}{2}+52x\right]_2^4$$

$$\Rightarrow\frac{1}{3}\left[13*2-13*2-\frac{13}{2}+13\right]$$

$$+\frac{1}{6}\left[-13x8+52x4+13x2-52x2\right]$$

$$\Rightarrow\frac{13}{2}+\frac{26}{6}=\frac{\cancel{39}^{13}}{\cancel{6}_2}=\frac{13}{2}squnits$$

Q. 23. Let A =R x R and * be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d)

Show that * is commutative and associative. Find the identify element for* on A. Also find the inverse of every element (a, b) E A.

Example Let $A = N \cup \{0\} xN \cup \{0\}$ *and let* '*'

be binary operation on A defined by

$(a,b)*(c,d) = (a+c, b+d)$ *For all* $(a,b),(c,b) \in A.$

Show that

(i) '*' is commutative on A.

(ii) '*' is associative on A.

Also find the identity element, if any, in A.

SOLUTION (i) Commutativity: Let (a,b), (c,d)A. $\in$ Then,

$\because (a,b)*(c,d) = (a+c, d+b) and (c,d)*(a,b) = (c+a, d+b)$

$\because a+c = c+a \ \ and\ b+d = d+b \qquad for\ all\ a,b,c,d \in N$

$\Rightarrow (a,c)*(c,d) = (c,d)*(a,b) \qquad for\ all\ (a,b),(c,d) \in NxN = A$

$\Rightarrow$ '*' *is Commutative onA.*

Please Refer RD Sharma page 3.20 Example 9

Q. 24. Three numbers are select at random (Without replacement) from first six positive integers. Let X denotes the largest of the three numbers obtained. Find the probability distribution of X. Also. Find the mean and variance of the distribution.

Ans. we observe that X on take value 3, 4, 5 6.

P (X=3)=Probability that largest of three number is 3.

$$=\frac{3}{6}X\frac{2}{5}X\frac{1}{4}*3$$

P (x=4)= Probability that largest of 3 no. is 4

$$=\frac{4}{6}X\frac{3}{5}X\frac{2}{4}*{}^{4}c_{3}$$

$P(x=5)=\Pr obalitity\, that\, l\arg est$
$of\ 3 three\, no\, is 5$

$$=\frac{5}{6}*\frac{4}{5}*\frac{3}{4}*{}^{5}c_{3}$$

$P=(x=6)={}^{6}c_{3}$

X	3	4	5	6
P(X)	$\frac{3}{20}$	$\frac{4}{5}$		

To Be solved later…..

Q. 25. A retired person wants to invest an amount of Rs. 50,000 his broker recommends investing in two types of bonds. A and B yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs. 20, 000 in bond A and at least R.s 10, 000 in bond B. he also wants to invest at least as much in bond A as in bond B solve this linear programming problem graphically to maximize his returns.

$Q.25.\ Let\, his\, investment\, on\, Bond\, 'A' = Rs.x$

$\underline{\hspace{6cm}}\ Bond\, 'B' = Rs.Y$

$$x + y \le 50,000$$
$$x \ge 20,000$$
$$y \ge 10,000$$
$$z = \frac{10}{100}x + \frac{9}{100}y$$

Q. 26. Find the equation of the plane which contains the line of intersection of the planes.

r. (i-2j+k) -4= and

r. (-2i +j+k)+5=0

And whose intercept on x- axis is equal to that of on Y-axis.

Sol. 26 $\begin{matrix} x - 2y + 3z = 4 \\ -2x + y + z = -5 \end{matrix}$

equation of intersection of planes

$$x - 2y + 3z - 4 + \lambda(-2x + y + z + 5) = 0$$

$$(1-2\lambda)x + (-2+\lambda)y + (3+\lambda)z = 4-5\lambda$$

$$\frac{x}{\frac{4-5\lambda}{\lambda-2}} + \frac{y}{\frac{4-5\lambda}{\lambda-2}} + \frac{z}{\frac{4-5\lambda}{\lambda+3}} = 1$$

$$\left(\frac{\cancel{4-5\lambda}}{1-2\lambda}\right) = \left(\frac{\cancel{4-5\lambda}}{1-2\lambda}\right)$$

$$\lambda - 2 = 1 - 2\lambda$$
$$3\lambda = 3$$
$$\lambda = 1$$